Bounding the Error of Path Loss Models

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***Abstract*—In this paper we analyze the efficacy of basic path loss models at predicting median path loss in urban environments. We attempt to bound the practical error of these models and look at how they may hinder practical wireless applications, and in particular dynamic spectrum access networks. This analysis is made using a large set of measurements from production networks in two US cities. We are able to show quantitatively what many experienced radio engineers understand: these models perform poorly at predicting path loss in even relatively simple outdoor environments and are of little practical use aside from making crude estimates of coverage in the least demanding applications. As a solution, we advocate a renewed focus on measurement-based, adaptive path loss models built on appropriate statistical methods.**

I. Introduction

Today, as "white-spaces" dynamic spectrum access (DSA) networks are being planned, propagation path loss models are gaining renewed attention. These next-generation networks put great faith in the accuracy of propagation models—using them to determine boundaries for their transmissions ("geo-fencing") and in decisions about when and where it is safe to transmit [1]. Errors in prediction here can lead to substantial misinterpretation of spectrum occupancy, which can result in harmful interference with primary users or limit spectrum reuse, leading to reduction in system performance. Bounding the accuracy of these models is essential to determining the feasibility and capacity of future DSA networks [2]. In addition to their application to next-generation networks, path loss models have great importance for traditional wireless networks in terms of network planning. And, once a network is built, in identifying and repairing coverage gaps [3]. In both cases, understanding accuracy is crucial to the success of these applications.

There is no shortage of path loss model proposals in the literature. Yet, despite the large quantity (and variety) of work done, we recognize an important shortcoming that we begin to address in this work: there have been relatively few comparative evaluations of models using a sufficiently representative dataset as a basis for evaluation. And, those studies that do exist make comparisons between a small number of similar models and do not attempt to put *practical bounds on the error of these models*. While there has been substantial work in certain frequency ranges, for instance in the VHF band where solid work in the 1960's produced well validated results for analog television (TV) propagation, it is not clear how well these models work for predicting propagation for different types of systems operating at different frequencies. The result is that wireless researchers and engineers are left without proper guidance in picking among dozens of seemingly equivalent models where the ill-effects of using a model outside of its intended domain are not well established.

In this paper, we describe, implement, and analyze *30 propagation models spanning 65 years of publications*. Our focus is the efficacy of these models at predicting median path loss values in urban environments. Although many of these models are greatly different from one another, they all make use of the same basic variables on which to base their predictions: position (including height and orientation) of the transmitter and receiver, carrier frequency, and digital elevation model and land cover classification along the main line-of-sight (LOS) transmit path. In the present study, we are not including ray-tracing models (e.g., [4]) or partition based models (e.g., [5]) that require substantial knowledge of the environment, which is seldom available, and rarely at the precision required to make useful predictions. We are also not considering active-measurement models (e.g., [6]) that make use of in-situ measurements to correct their predictions. We will analyze more dynamic models in later work.

To perform our evaluation we use a large set of unique active and passive measurements collected in two US cities, involving three distinct measurement campaigns. These measurements paint a detailed picture of the practical propagation environment of typical systems operating in the 2.4 GHz ISM band in urban environments. We also make use of well-known publicly available data collected at 900 MHz as a basis for comparison. This work extends our related work that analyzed the efficacy of path loss models in "simple" *rural* environments [7]. In that work, we found substantial error in the predictions of commonly used path loss models. Indeed, many authors have considered the problem of predicting outdoor path loss to be solved. We will see this is far from true—making accurate *a priori* predictions about path loss, without in-situ measurements, is a very difficult task even in "simple" environments with the models available.

In the end, our results show that no single model is able to predict path loss consistently well. Models that perform well in one circumstance do not perform well in others, and the best performing models cannot be sufficiently tuned to a datum such that they are any better than basic fits to a small number of measurements.

II. RELATED WORK

The vast majority of existing work analyzing the efficacy of path loss models has been carried out by those authors who are proposing their own improved algorithm. In such cases, the authors often collect data in an environment of interest and then show that their model is better able to describe this data than one or two competing models. Unfortunately, this data is rarely published to the community, which makes comparative evaluations impossible. One noteworthy exception is the work of the COST-231 group in the early 1990's, which published a benchmark dataset (900 MHz measurements taken in European cities) and produced a number of competing models that performed well with respect to this reference [8].

Similarly, there was substantial work done in the US, Japan, and several other countries in the 1960s and 1970s to come up with accurate models for predicting the propagation of analog TV signals (e.g., [9]). This flurry of work produced many of the models that are still used today in network simulators and wireless planning tools: the Longley-Rice Irregular Terrain Model (ITM) [10], the Egli Model [11], and the Okumura-Hata model [12], to name a few. However, it is unclear what the implications are of using these models, which were created for use in a specific domain, to make predictions about another domain.

There are several studies similar to our own, that compare a number of models with respect to some data. In [13], the authors compare five models with respect to data collected in rural and suburban environments with a mobile receiver at 910 MHz. They discuss the abilities of each model, but abstain from picking a winner. In [14], the authors compare three popular models to measurements collected at 3.5 GHz. In that work, which compares a least-squares fit of measurements to the model predictions, the authors highlight the best of the three, which turns out to be the ECC-33 model proposed in [15]. In [16], Sharma et al. do a very similar analysis, but instead focuses on measurements made in India at 900 and 1800 MHz. In contrast to [14], they find that the SUI and COST-231 models perform best. We believe our work here is the first to do an in-depth and rigorous analysis of a *large number of diverse propagation models* using a *large and realistic dataset from a production network*. And, it is the first such comparative study looking at results for the widely used 2.4 GHz ISM band.

III. MEASUREMENT

In this work, we analyze the performance of various path loss prediction algorithms using measurements from three distinct campaigns that combine to form a cohesive picture of the urban wireless propagation environment. Figure 1 provides a schematic of the three data sets and Table I provides further details. Our three campaigns cover the three transceiver configurations that we see as most important in the urban wireless environment. The first, **A**, concerns well-positioned (i.e., tower or rooftop) fixed wireless transceivers. This sort of link is typically used for back-haul or long distance connections (e.g., [17]). The second, **B**, concerns propagation between a single fixed ground-level node (i.e., on a utility pole) and mobile ground-level client devices. Finally, **C**, concerns infrastructure network configurations where one fixed well-positioned transmitter (access point) is responsible for serving multiple ground-level mobile nodes.

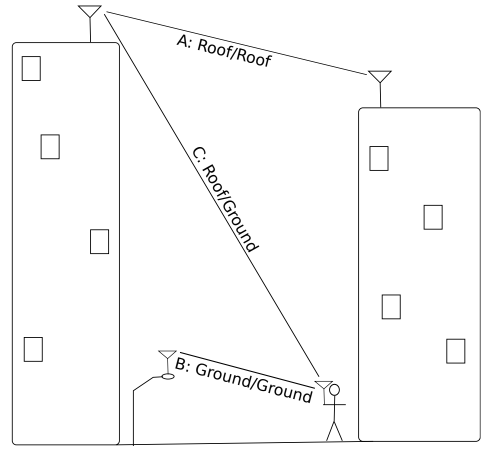


Fig. 1: Visual schematic of three urban datasets. A: roof to roof measurements from CU WART (Wide Area Radio Testbed), B: ground (light-poles) to ground (mobile node) measurements in Portland, Oregon, C: roof to ground and ground to roof measurements from CU WART.

With the exception of the COST-231 data, all data sets were collected using commodity hardware and packet-based measurements were used to determine received signal strength. This approach differs from some prior work on path loss modeling that uses continuous wave (CW) measurements [2], [8]. In prior work we have shown that it is possible to calibrate commodity hardware so that it is capable of making measurements with sufficient accuracy for modeling path loss [18]. However, packet-based methods necessarily "drop" measurements for packets that cannot be demodulated. Without driver modification, they also update noise-floor measurements infrequently. For the purpose of analyzing accuracy of median path loss prediction, these limitations of our dataset are not problematic. However, it should be noted that packet-based measurement methods are not appropriate for all modeling tasks—the tradeoff between convenience and affordability of commodity hardware versus the completeness of the measurements must be considered.

A. Back-haul

Our first data set, **A**, was collected using the University of Colorado (CU) Wide Area Radio Testbed (WART), which is composed of six 8-element uniform circular phased array antennas [19]. Figure 2 shows the layout of this testbed. The devices are mounted on roof-tops on the CU campus and in the surrounding city of Boulder, Colorado. These devices can electronically change their antenna pattern, which allows for them to operate as a directional wireless network with a main-lobe pointed in one of 16 directions or as an omni-directional

Campaign	Name	Environment	Type	Sites	Measurements
A	wart	Campus	Point-to-Point	7	33,881
A	wart/snow	Campus	Point-to-Point	7	24,867
B	pdx	Urban	Urban Mesh/Infrastructure	117	117
B	pdx stumble	Urban	Urban Mesh/Infrastructure	59,131	200,694
C	boulder/ptg	Campus	Infrastructure/Downstream	1,693	1,693
C	boulder/gtp	Campus	Infrastructure/Upstream	329	329
D	cost231	Urban	Infrastructure/Downstream	2,336	2,336

TABLE I: Summary of Data Sets

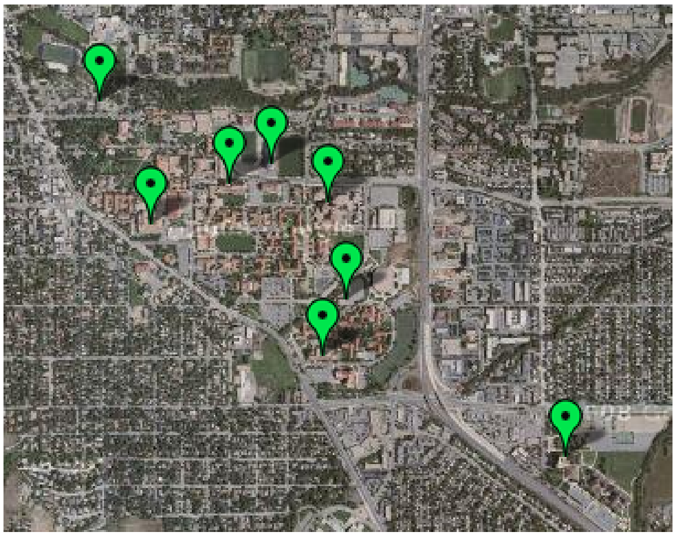


Fig. 2: University of Colorado Wide Area Radio Testbed.

antenna whose gain is (approximately) uniform in the azimuth plane. To collect this data, an "NxN scan" is done of the sort proposed in [20], which results in Received Signal Strength (RSS) measurements for every combination of transmitter, receiver, and antenna pattern. In short, this works by having each AP take a turn transmitting in each state while all other nodes listen and log packets. Identical measurements were collected during the winter (no leaves), during a snowstorm, and during the summer of 2010. These network measurements are applicable to any rooftop-to-rooftop type communication system, including cell networks, and point-to-point or point-to-multipoint wireless back-haul networks both with directional antennas and with omnidirectional antennas.

B. Street Level Infrastructure

Our second set of measurements, **B**, comes from a (now defunct) municipal wireless mesh network in Portland, Oregon. In this network, 70 access points are deployed on utility poles in a 2 km by 2 km square region. Each access point has a 7.4 dBi omnidirectional antenna that provides local coverage in infrastructure mode. These measurements were collected during the summer of 2007. This data set, which consists of both laborious point-testing and extensive war-driving data is most representative of ground-to-ground links in urban environments. The point-testing data was collected at 117 sites selected uniformly at random within the bounding box. At each site, a fixed point tester logged GPS coordinates, received signal strength, and attempted to associate with the network and perform application layer tests. More detail on this point testing procedure is described in [21]. The war-driving data was collected by driving on each publicly accessible street in the coverage region. Figure 3 shows the layout of this network and the measurements from the war-driving dataset. To compress this data set slightly, we truncate the precision of GPS coordinates to five significant digits, which has the effect of grouping data points within a 0.74 m circle.

C. Wide Area Infrastructure

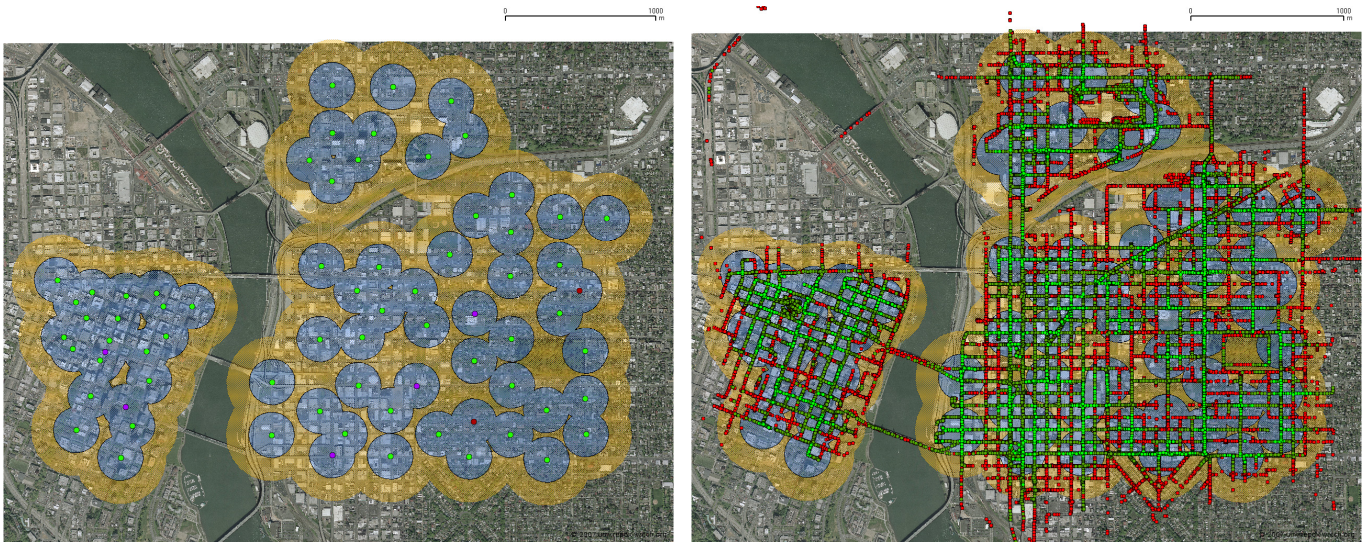
The final data set, **C**, was collected using a mobile node (a Samsung brand "netbook") with a pair of diversity antennas. In this experiment, the 6 rooftop CU WART nodes were configured to transmit 80 byte "beacon" packets every $0.5 + U(0.0, 0.5)$ seconds where $U(X, Y)$ is a uniformly distributed random number between X and Y. Beacons are configured to transmit at 1 Mbps, so that possible effects of Doppler spread on higher datarate waveforms are avoided. Similarly, the mobile device was configured to transmit beacons at the same rate. Meanwhile, each rooftop testbed node was configured to its 9 dBi omnidirectional antenna pattern. All nodes, including the mobile node were configured to log packets using a second monitor-mode (promiscuous) wireless interface. The mobile node was additionally instrumented with a USB GPS receiver that was used both to keep a log of position and to synchronize the system clock so that the wireless trace was in sync with the GPS position log. These measurements were collected during the summer of 2010. During the experiment, the mobile node was attached to an elevated (nonconducting) platform on the front of a bicycle. The bicycle was pedaled around the CU campus on pedestrian paths, streets, and in parking lots. This data set is most representative of an infrastructure wireless networks where a well-positioned static transmitter must serve mobile clients on the ground. We subdivide this data set into the upstream part and the downstream part.

D. COST-231 Data

In addition to these three new sets of measurements, we also make use of a single reference data set from the literature collected by the COST-231 group at 900 MHz [8] in Munich in 1996. This data set, which provides path loss measurements collected by a mobile receiver from three well-placed (rooftop) transmitters is closest in intent to our data set **C**, but does not include upstream (mobile transmitter, fixed receiver) data as ours does.

Name	Short-Name	Custom Parameter Settings	Category	Coverage Domain	Cite	Year
Friis Freespace	friis	$\alpha = \{2,4,6\}$	Foundational	$d > 2a^2/\lambda$	[22]	1946
Egli	egli		Basic	$30MHz < f < 3GHz$	[11]	1957
Hata-Okumura	hata	$size = \{o,s,m,l\}$	Basic	$1\,km < d < 10\,km; 150 <= f <= 1500\,MHz$ $30 <= h_1 <= 200\,m; 1 <= h_1 <= 20$	[12]	1968
Edwards-Durkin	edwards	$\Delta h = \{0,15,200,400\}$	Basic/Terrain		[23]	1969
Allsebrook-Parsons	allsebrook	$\Delta h = \{0,15,200,400\}$; $h_0 = 30, d^2 = 5$	Basic/Terrain	$f \in 85, 167, 441\,MHz$; Urban	[24]	1977
Blomquist-Ladell	blomquist	$\Delta h = \{0,15,200,400\}$	Basic/Terrain		[25]	1977
Longley-Rice Irregular Terrain Model (ITM)	itm	$c = \{5,6\}; \epsilon = \{5,15\}$; $\sigma = \{0.001, 0.005\}$	Terrain	$1 < d < 2000\,km$ $0.02 < f < 20\,GHz$	[10]	1982
Walfish-Bertoni	bertoni	$h_0 = 20; w = 10$	Foundational		[26]	1988
Flat-Edge	flatedge	$n = \{1,2,5,10,20\}; h_0 = 20; w = 10$	Basic		[27]	1991
TM90	tm90		Basic	$d <= 10 miles; h_1 <= 300 feet$	[28]	1991
COST-Hata/Cost-231	cost231	$size = \{m,l\}$	Basic	$1 < d < 20\,km$;	[8]	1993
Walfisch-Ikegami	walfish	$size = \{m,l\}; b = \{20,30\}; w = 10$	Basic	$0.2 < d < 5\,km$; $0.8 < f < 2\,GHz$; $4 < h_b < 50\,m$; $1 < h_m < 3\,m$	[8]	1993
Two-Ray (Ground Reflection)	tworay		Foundational		[29]	1994
Hata-Davidson	davidson	$size = \{o,s,m,l\}$	Basic	$1 < d < 300\,km$; $0.15 < f < 1.5\,GHz$; $30 < h_b < 1500\,m$; $1 < h_m < 20m$	[30]	1997
Oda	oda		Basic		[31]	1997
Erceg-Greenstein	erceg	$terrain = \{A,B,C\}$	Basic	$f \approx 1.9\,GHz$; Suburban	[32]	1998
Directional Gain Reduction Factor	grf	$season = winter$	Supplementary	Dir. Recv. Ant.,$f \approx 1.9\,GHz$	[33]	1999
Rural Hata	rural.hata	$env = rural$	Basic	$f \in 160, 450, 900\,MHz$; Rural (Lithuania)	[34]	2000
ITU Terrain	itu		Terrain		[35]	2001
Stanford University Interim	sui	$terrain = \{a,b,c\}$	Basic	$2.5 < f < 2.7\,GHz$	[15]	2001
Green-Obaidat	green		Basic		[36]	2002
ITU-R/CCIR	itur	$b\% = \{25,50,75\}$	Basic	$1km < d < 10km$; $1.5 < f < 2\,GHz$; $30 < h_b < 200\,m$; $1 < h_m < 10\,m$	[35]	2002
ECC-33	ecc33	$size = \{m,l\}$	Basic	$1 < d < 10\,km; 700 <= f <= 3000\,MHz$ $20 <= h_1 <= 200\,m; 5 <= h_1 <= 10\,m$	[37]	2003
Riback-Medbo	fc		Supplementary	$0.46 < f < 5.1\,GHz$	[38]	2006
ITU-R 452	itur452		Terrain		[39]	2007
IMT-2000	imt2000		Basic	Urban	[40]	2007
deSouza	desouza	$hum. = \{53,72,80\}$	Basic	$f \approx 2.4\,GHz; d < 120\,m$	[41]	2008
Effective Directivity Antenna Model	edam	$env. = urban\ outdoor$	Supplementary	Directional Antennas; $f \approx 2.4\,GHz$	[42]	2009
Herring ATG/GTG	herring		Basic	$f \approx 2.4\,Ghz$	[43]	2010

TABLE II: Models Studied along with their categorization, custom parameter settings (where applicable), coverage domain (when available), citations, and year of (earliest found) publication.



(a) APs and Assumed Coverage

(b) Measurements

Fig. 3: Portland MetroFi network with operator-assumed coverage. Lighter dots (green) indicate stronger signal. 500 ft. and 1000 ft. radii circles are placed around each AP to show the operator assumed coverage.

IV. Models

Table II provides details of the models evaluated in this study. In the following subsections we will briefly discuss each major category of model within our proposed taxonomy and list notable examples. Due to space constraints we are unable to discuss each model that we implement and instead focus on describing the most prevalent themes: Theoretical Models, Basic Models, Terrain Models, Supplementary Models, and Advanced Models.

At a high level, a model's task is to predict the value of $L + S$ in this equation:

$$P_r = P_t - (L + S + F) \tag{1}$$

where P_r and P_t are the received and transmitted powers and the total path loss $(L + S + F)$ is the sum of the free-space path loss (L), the loss due to shadowing/slow-fading (S, i.e., large fixed obstacles like mountains and buildings), and F the small-scale/fast fading due to destructive interference from multipath effects and small scatterers. Models cannot, without perfect knowledge of the environment, be expected to predict the quantity F. In most applications, this additional error is computed from a probability distribution (often Raleigh, although Rician and m-Nakagami are popular). For the protocols used in our study this quantity tends to be small due to the averaging effect of wide-band modulation schemes and explicit averaging from multiple measurements [44]. The error that does remain can be be quantified (by looking at the variance in measurements on a fixed path) and then accounted for in analysis.

It is worth noting that among the models we have implemented, very few were designed for exactly the sort of networks we are studying. Indeed, some are very specific about the type of environment in which they are to be used. In this work we do not strictly adhere to these coverage requirements because we observe that they are not largely followed in the literature (the Longley-Rice Irregular Terrain model, in particular, is frequently used well outside of its intended coverage). In this study both appropriate and "inappropriate" models are given an equal chance at making predictions for our network. We have no starting bias about which should perform best.

A. Theoretical/Foundational Models

The first models worth considering are purely analytical models derived from the theory of idealized electromagnetic propagation. Although these models are questionably accurate, they are simple to understand and implement and as a result they have been widely adopted into network simulators and other applications and often function at the center of more complex models. Important examples include Friis' equation for free space path-loss between isotropic transmitters [22] and the two-ray ground-reflection model [29], [45]. Friis equation is an integral component of many of the more complex models. It observes that power is attenuated in free-space proportional to the distance squared:

$$P_r/P_t = \left(\frac{\lambda}{4\pi d}\right)^2 \tag{2}$$

which provides the ratio between received power (P_r) and transmitted power (P_t) as a function of distance (d) and wavelength (λ). More commonly this equation is given in the logarithmic domain:

$$P_r = P_t - (20log_{10}(d) + 20log_{10}(f) + 32.45) \quad (3)$$

Where distance (d) is given in km, carrier frequency (f) in MHz and power is in units of decibels relative to a mW (dBm).

B. Basic Models

The models that we call "basic" models, are the most numerous. They compute path loss along a single path and often use corrections based on measurements made in one or more environments. In general, they use the distance, carrier frequency, and transmitter and receiver heights as input. Some models also have their own parameters to select between different modes of computation or fine tuning. Here we subdivide these models into deterministic and stochastic. The stochastic models use one or more random variables to account for channel variation (and hence are able to predict a distribution instead of a median value). The Egli model [11], [13], [46], Green-Obaidat [36], Hata-Okumura [46], [12] (and its many derivative models [40], [8], [47], [34]), and the Walfish-Ikegami model [48] are good examples of deterministic basic models. Stochastic models include the recent Herring models [43] and the Erceg models [32], [15] among others. Because we are concerned with predicting median path loss, we disable the stochastic element of these models and simply use their median prediction.

C. Terrain Models

Terrain models are similar to the basic models, but also attempt to compute diffraction losses along the line of sight path due to obstructions (terrain or buildings, for instance). They are an order of magnitude more complex, but are immensely popular especially for long propagation distances at high power in the VHF band (i.e., television transmitters). Important examples include the ITM [10], [49], which is widely used in propagation planning software (e.g., [50], [51]), the ITU-R 452 model, which is quite similar with some added complexities [39], and the straight-forward ITU-Terrain Model [45], [35].

D. Supplementary Models

Supplementary models cannot stand on their own, but are instead intended to make corrections to existing models. These models are best subdivided into the phenomenon they are wishing to correct for: stochastic fading [52], [53], [46], frequency [38], atmospheric gases [54], terrain roughness [13], and antenna directivity [33], [42] cover the majority of models. When appropriate, we use these models to correct the other models (i.e., frequency correction for the Hata model (hata.fc), or directivity correction for the CU-WART measurements).

E. More Advanced Models

There are also two major categories of models that we are not considering in this study: many-ray (ray-tracing) models and active-measurement models. Although to some extent these models typify the state of the art with respect to propagation modeling, they are not the models that are widely used in simulators and propagation planning tools. To a large extent, this is because they have greater data requirements. Many-ray models require high-resolution data describing the environment and substantial computation time. These predict the summed path loss along many paths by uniform theory of diffraction (or similar) [55], [56], [4].

Active-measurement models take the perspective that the only way to make realistic predictions is to marry an *a priori model* with in-situ measurements. The development of these models are fairly immature but there are front-runners, including the proposal of Robinson et al. in [3]. A related set of "partition" models, most well known in indoor propagation applications, combines the multi-ray approach with some direct measurement of losses due to obstacles [5].

F. Implementation

In our implementation, each of the 30 models is implemented from their respective publications in the ruby programming language. Only one of the models, the ITM [10], has a reference implementation. Hence, there are fundamental concerns about correctness. To address this, we perform basic sanity checking of model output. However, without access to the data sets on which the models were derived, or reference implementations, we are unable to make a more rigorous verification than this.

Terrain Models require access to a Digital Elevation Model (DEM), and in the case of ITU-452 a land-cover database (LCDB) as well. The DEM we use is publicly available raster data from the United States Geological Survey (USGS) Seamless Map Server, providing 1/3 arc-second spatial resolution. The LCDB is also provided by the USGS as a raster dataset, which is generated by the USGS using a trained decision tree algorithm. We use the GDAL library [57] to perform coordinate conversions and data extraction to generate path profiles for the terrain algorithms.

Our evaluation proceeds as follows: for each link in each data set, we use the various algorithms to make a path loss prediction, using one or more custom parameter configurations where applicable. If it is unclear which parameter settings are most appropriate, we try a range of reasonable parameters, which are listed in table II. This requires a substantial amount of computation, but is trivially parallelizable. To make the computation of results tractable, we subdivide the task of prediction into a large number of simultaneously executing threads and merge the results after completion. This must occur in two sequential stages. During the first stage, path profile information is extracted and prepared for each link in parallel, and during the second stage this information is fed to each algorithm for each link, which can also be done in parallel. With the merged data in hand, each prediction is compared with an oracle value for the link. This oracle value is computed from the measured received signal strength for the link as well as known values for the transmitter power and antenna gain. For omnidirectional nodes, a static antenna gain term is assumed. For directional nodes (i.e., the phased array directional patterns), measured antenna patterns are used. With both oracle and predicted values in hand, computing the error is simply a matter of finding the difference.

V. RESULTS

We begin by explicitly fitting our data to a theoretical model and looking at the number of measurements required for a fit. This gives us an initial estimate of expected error for direct (naïve) fits to the collected data. Then, to analyze the performance of the algorithms, we propose five metrics of decreasing stringency. In the following subsections, we will discuss the results with respect to these metrics as well as general trends and possible sources of systematic error. Finally, in an attempt to put a lower bound on model error, we engage in explicit parameter fitting of the best models and compare this best-case performance to the naïve approach of straight-line fitting.

A. Explicit Power law Fitting

Consider equation 3 in section IV-A, which describes the fundamental power law relationship between path loss and distance. It is common in the literature to show this relationship as a straight line on a log/log plot. If we modify this equation to have a flexible exponent and error term, it is possible to do a linear fit in the log/log domain and come up with empirical estimates of the exponent (α) and offset (ϵ):

$$P_r = P_t - (\alpha 10 log_{10}(d) + 20 log_{10}(f) + 32.45 + \epsilon) \quad (4)$$

Figure 4 shows the Boulder downstream (boulder/ptg) and COST-231 data as examples of 2.4 GHz and 900 MHz measurements. One unavoidable side-effect of packet-based measurements is that it is impossible to record SNR values for packets that fail to demodulate. Hence, because the 2.4 GHz data is derived from packet-based measurements, low SNR values (and therefore high path loss values) are under-represented here, which leads to "shallow" fits and unrealistically low values of α. Additionally, those packet-based measurements that are received, report an SNR which is computed from the packet preamble and is effectively an average over the width of the 20 MHz Direct Sequence Spread Spectrum (DSSS) channel, whereas the COST-231 measurements are narrow-band measurements centered at a specific frequency. As a result, while it is safe to make comparisons between the 2.4 GHz datasets, it is not safe to directly compare the slope of the 900 MHz and 2.4 GHz fits.

Fits are computed using linear least square regression. Table III lists fitted parameters (α, ϵ) and residual standard error (σ). Between the 2.4 Ghz datasets, we can see that there is little consensus about the slope or intercept of this power law relationship, except that it should be in the neighborhood of $\alpha \approx 2$ and $\epsilon \approx 15$. All fits are noisy, with standard error around 8.68 dB on average. This residual error tends to be Gaussian, which is also in agreement with previously published measurements (e.g., [29]). However, the size of this error is almost two orders of magnitude from the 3 dB, that Rizk et al. suggest as an expected repeated measures variance for outdoor urban environments (and hence the expected magnitude of the error due to temporally varying fast-fading) [58]. Looking at figure 4, it is easy to see that the 2.4 GHz measurements are substantially less well-behaved than the COST-231 data, even in comparable environments. It stands to reason that this would be the case, since higher frequency transmissions are more readily absorbed by obstacles.

In order to understand how many measurements are needed to create a fit of this sort, we take successively increasing random samples of the datasets and use these subsets to generate a fit. We then look at how the residual error of the model (with respect to the complete dataset) converges as the sub-sample size increases. Figure 4c shows this plot for the CU-WART (data set A) measurements as an example, although all plots follow a similar trend: the eventual model is closely matched with approximately 20, or at most 40, data points. Table III gives an approximate minimum sample size for each data set in the column labeled N.

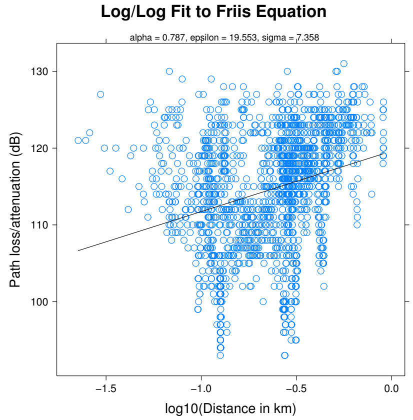
B. Five Metrics

To analyze the performance of the models, we propose five metrics. To clarify the plots, we only show results from the 18 best performing model configurations. Figure 5 shows the performance of these models for making predictions about all of our links and a detailed description follows.

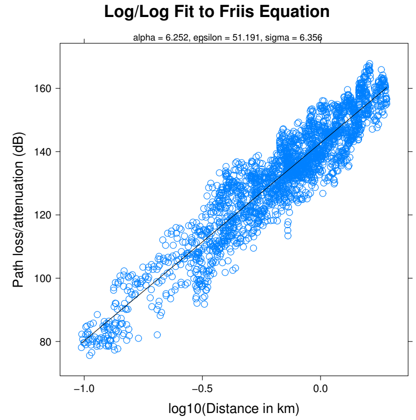
The first metric, which is the most straight-forward is overall Root Mean Square Error (RMSE)[1]. We also show the spread-corrected RMSE (SC-RMSE), where the measured spread (standard deviation) is subtracted from the RMSE. This is effectively a way of showing the "best case" RMSE for the model in terms of the expected magnitude of small scale temporal channel variation. We can see that the best models achieve an RMSE on the order of 10 dB, and the worst (of the best) approach more than 50 dB. The overall winners are the Hata model, the Allsebrook model, the Flat Edge model, and the ITU-R model. This follows from expectations because all of these models were derived for predicting path loss in urban environments. The Hata model and Allsebrook models are based on measurements from Japanese and British cities respectively. The Flat Edge model is a purely theoretical model based on the Walfisch-Bertoni model, which computes loss due to diffraction over a set of uniform screens (simulating buildings separated by streets).

Our next metric is a competitive definition of success: for what percentage of links does a given model make the best prediction. Figure 5b gives this result as the leftmost of three bars for each model. We can see, rather significantly, there is no one clear winner, with the most successful models sharing approximately 10-15% of the winnings. The other two bars in this figure are our third metric, an individualistic definition of success: the percentage of predictions that are within one (or two) standard deviations of the correct median value. The best performing models (Allsebrook, Flat Edge, Herring

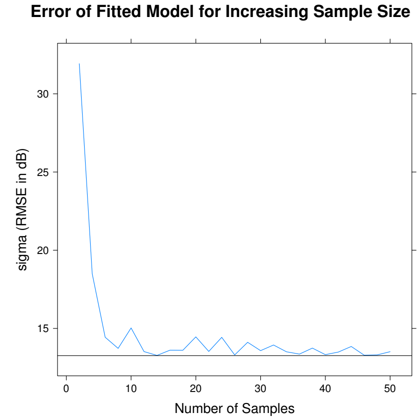
[1]For all intents and purposes, standard error (σ) and RMSE are interchangeable.



(a) Boulder Measurements and Fit



(b) COST231 Measurements and Fit

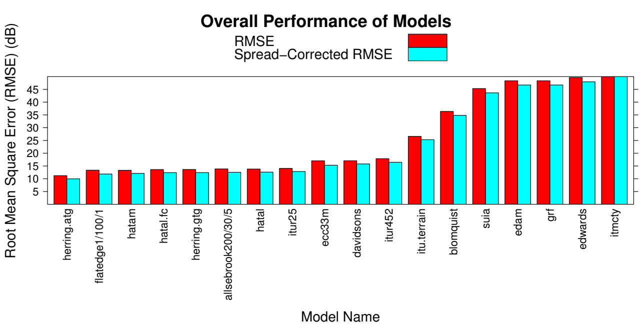


(c) Minimum Sufficient Sample Size

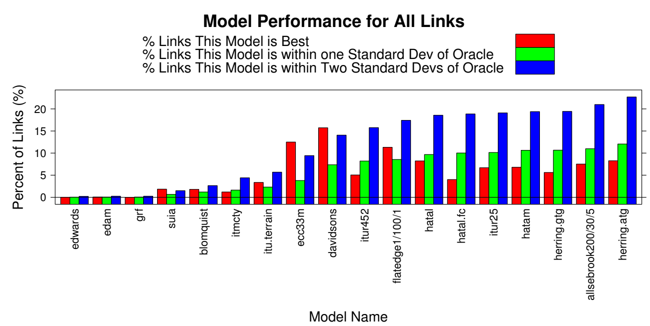
Fig. 4: Explicit Power law fits to Data. Plot (a) and (b) show data long with fits and parameters. Plot (c) shows the RMSE of a fit to the CU-WART data using an increasing number of sample points. A horizontal line is placed at the RMSE achieved with a fit to the entire data.

Name	α	ϵ	σ	N	Top Three Performing Models by SC-RMSE						Ideal RMSE
wart	1.86	9.05	13.26	15	flatedge	13.73	itu.terrain	13.89	hatao	14.03	1.96
wart/snow	1.92	9.25	13.36	15	itu.terrain	13.93	flatedge	14.16	hatao	14.19	1.87
pdx	2.25	19.53	7.8	5	allsebrook200	8.38	hatal	8.97	davidsons	9.37	1.14
pdx stumble	1.79	27.08	8.96	40	allsebrook400	8.34	itur25	10.50	hatam	10.51	1.02
boulder/ptg	0.79	19.56	7.36	20	allsebrook400	7.90	ecc33m	9.38	hatam	10.47	0.94
boulder/gtp	0.27	10.88	3.67	5	allsebrook400	5.45	hatal.fc	7.15	edwards200	8.51	1.01
cost231	6.25	51.19	6.36	15	edwards200	9.23	hatam	9.99	itur25	10.55	1.23

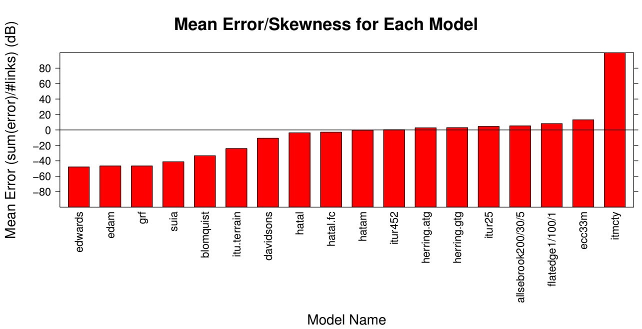
TABLE III: Summary of results by dataset



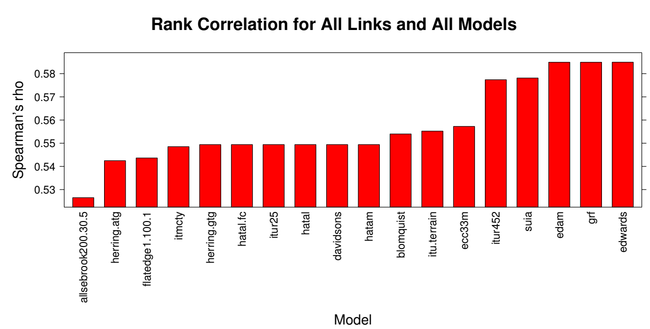
(a)



(b)



(c)



(d)

Fig. 5: Overall performance of each model for each metric for all environments.

Air-to-Ground, and ITU-R) score between 10% (for within one standard deviation) and 20% (for within two standard deviations) on this metric.

Our fourth metric is skewness, which is the overall sum of prediction error scaled by the number of predictions. We observe that some applications may have a particular cost/benefit for under or over-predictions. Models that systematically over-predict path loss (and therefore under-predict received signal strength) score a high value on this metric. Models that systematically under-predict, score a large negative value. And, models that make an equal amount of under and over-predictions will score a value of zero. We can see that some models are more skewed in their predictions. The best performing models by this metric are the ITU-R 452 and Hata.

Our final metric is rank correlation using Spearman's ρ[2]. In some applications, predicting an accurate median path loss value might not be necessary so long as a model is able to put links in a correct order from best to worst (consider, for instance, the application of dynamic routing). Spearman's ρ is a non-parametric measure of statistical dependence and in this application describes the relationship between ranked predictions and oracle values using a value between -1.0 (strong negative correlation) and 1.0 (strong positive correlation). All of our models score somewhere between 0.35 and 0.45, which indicate a moderate, but not strong, positive correlation. Interestingly, models that perform poorly in terms of overall error are among the best here (EDAM or Edwards-Durkin for instance), but the difference is not large enough to be considered significant.

Besides these results for all links combined, we also have studied the results for each data set. Table III lists the top three performing models (in terms of SC-RMSE) for each dataset. We can see that overall the Allsebrook-Parsons model performs very well, being in the top three for nearly all the data sets. Another big winner is the Hata model. We see almost an unmistakable correlation between model simplicity and performance. The Hata and Allsebrook models are among the most simplistic. The former is an empirical model developed from measurements in Japanese cities. The latter is a modified plane-earth model with empirical corrections from measurements in British cities and a terrain roughness parameter. However, there is not universal agreement between the data and there is certainly not agreement among our metrics.

One interesting additional observation from this data is that modeling path loss from directional transmitters is especially difficult. This can be seen in the fact that our data from the directional CU-WART testbed is particularly noisy. There have been several attempts to model this phenomenon explicitly in the past (e.g., [42], [33]), but we see that even using those, the error in prediction of directional propagation is still much greater than for omnidirectional transmitters.

[2]Kendall's τ would be an equally appropriate metric, but is slower to compute.

C. Factors Correlated with Error

In order to understand which variables may serve to explain model error, we performed a factorial analysis of variance (ANOVA) using spread-corrected error as the fitted value and transmitter height, receiver height, distance, line-of-sight (a boolean value based on path elevation profile), and dataset name. Although all of these variables show moderate correlations (which speaks to the fact that many models add corrections based on these variables), some are much better explanations of variance than others. Perhaps not surprisingly, distance and data-set name are the biggest winners with extremely large F-values[3](40,018 and 48,164 respectively). This leads to the conclusion that the best results can be obtained when an appropriate model is known for a given environment, and when the model is designed for the same distances of links being modeled. *Using models outside of their best-environment and best-distance coverage will result in substantial error*. This also motivates future work in hybridized models that change their approach based on the environment or length of links being modeled.

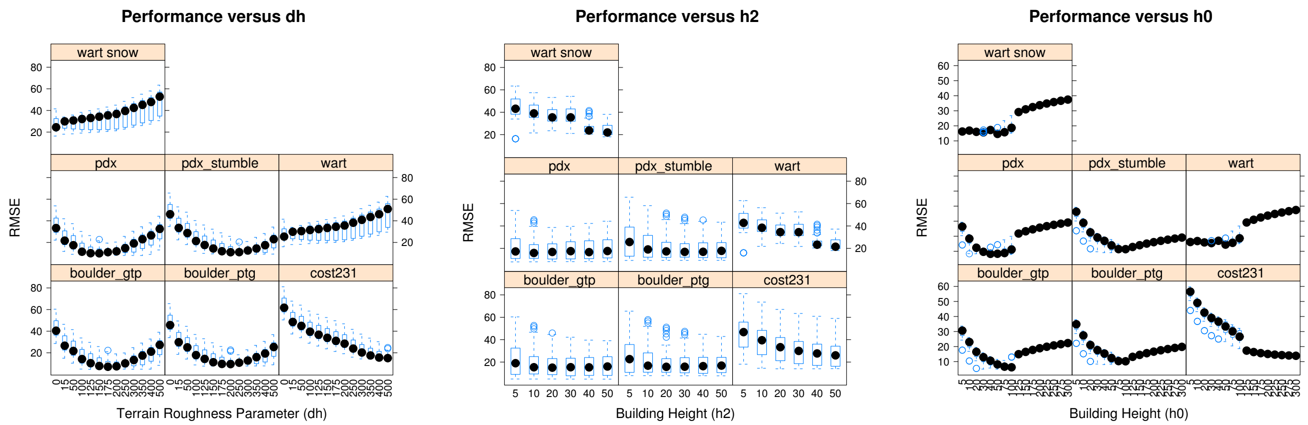
D. Explicit Parameter Fitting

In order to get an idea of minimum obtainable error with these models, we take two well performing models that have tunable parameters, Allsebrook-Parsons and Flat Edge, and proceed by searching the parameter space to find the best possible configuration. The Allsebrook-Parsons model takes three parameters (besides carrier frequency, which is common to nearly all the models): Δh, a terrain roughness parameter (in m), h_0, the average height of buildings (in m), and d_2, the average width of streets (in m). The Flat Edge model also takes three parameters: n, the number of buildings between the transmitter and receiver, h_0, the average height of these buildings (in m), and w, the street width (in m). After sweeping the parameter space, we use an ANOVA to determine the parameters that best explain the variance in the data.

We find that for the Allsebrook model, the Δh and h_2 parameters are both important and for the Flat Edge model, h_0 is the only significant parameter. Figure 6 shows the response (in terms of RMSE) for tuning these parameters. The optimal values can be determined from the minima of these plots and a similar approach could be carried out with any subset of our data. However, *the optimal parameters for one datum are not usually in agreement with others forcing a compromise in terms of accuracy and specificity*. Even with cherry-picked parameters, the RMSE is still in the neighborhood of 9-12 dB, which is too large for most applications.

If we consider 9 dB to be the minimum achievable error of a well-tuned model, it is interesting to note that approximately the same performance can be achieved with a straight-line fit through a small number ($\approx$ 20) of measurements as we did in section V-A. In [6], the authors found similar bounds on

[3]The F-value is a statistic that describes the ratio between explained variance and unexplained variance. Or, put differently, the ratio of between-group variability to within-group variability.



(a) (b) (c)

Fig. 6: Explicit parameter fitting for the Allsebrook and Flat Edge model parameters.

error (6-10 dB) attempting to fit a single model to substantial measurement data at 1900 MHz. If the domain of interest is network planning, and it is not possible to make measurements of a network (because it does not yet exist), then tuning an *a priori* model may be the right approach to take. However, if the goal is modeling the path loss of a network that can be directly studied, and taking 20 (randomly distributed) measurements is reasonably cheap, then this approach seems easy to advocate.

E. Practical Interpretation

As an example of what this means for real applications, consider figure 7, which shows a predicted coverage map for the Portland Metro-Fi network using two well-performing models tuned to their best performing configurations. We have also included versions of these maps with zero-mean 12 dB Gaussian noise, which approximates the expected residual error from these models. To generate these maps, the 2 km by 2 km coverage area was divided into a 500x500 raster and each pixel is colored based on predicted received signal strength, linearly interpolated between red (at -95 dBm) and green (at -30 dBm). For each pixel, we compute the predicted path loss from all 72 APs and the maximum value is used to color the pixel.

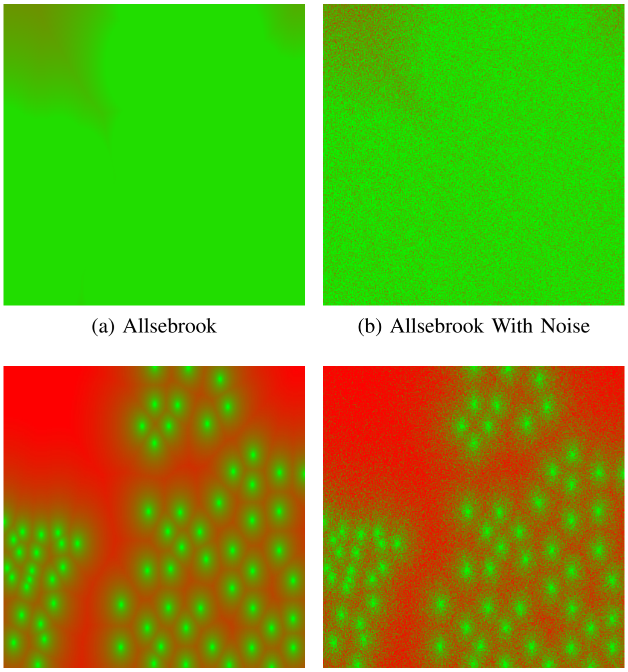
Comparing these maps to the empirical and operator-assumed coverage maps in figure 3, it is clear to see that there is no consensus on what the propagation environment looks like. The Allsebrook-Parsons model, which is well performing overall, and we have tuned to its best configuration, produces a map that is in stark disagreement with reality. The Hata model, on the other hand, may produce the picture that is closer to the measurements, but our results show that it is not the best performing model overall. Ultimately, the coverage map produced by the Hata model is little more than uniform propagation disks centered around the locations of access points.

If this were the Radio Environment Map (REM) that a DSA system used to predict propagation boundaries, there could be substantial problems as a result. In this scenario, we can imagine that red areas in the map are areas where it is safe for a secondary user to transmit. However, in cases where the REM underestimates signal propagation, substantial interference could occur with primary users. And, in cases where the signal is an overestimate, the secondary user will needlessly yield the channel, missing free spectrum opportunities and affecting performance. Based on these lackluster performance results, we cannot advocate the use of basic path loss models alone for interference prediction and REM computation.

Yet, the future holds promise. Consider the final column in Table III, which gives the RMSE for each dataset if we choose to take only the best prediction among all the predictions made by the 30 models and their configurations. This represents one version of a minimal achievable error in a world with a perfectly hybridized model that always knows which model to use when. In this scenario, we can see a very attactive bound on error—as low as 1 dB. We believe this indicates that there is still room for improvement. If we were able to determine the situations when each model is likely to succeed, then it is reasonable to assume that it is possible to construct a single hybrid model that is more accurate than the sum of its parts. In [7] we have shown that this approach can lead to modest improvements when hybridizing based on link length (distance), but a full exploration of hybridization is a worthwhile topic for future work.

VI. Conclusion

In this paper we have provided the first large scale investigation of the performance of path loss models at making predictions in urban environments. We based our analysis on a large corpus of measurements collected from production wireless networks in representative US cities. This work extends our work that looked at rural measurements and found similar results [7]. In sum, we feel that we have made a strong argument about the danger of using basic *a priori* models to predict the vagaries of the radio environment. We have



(a) Allsebrook (b) Allsebrook With Noise

(c) Hata (d) Hata with Noise

Fig. 7: Comparison of predicted coverage maps for Portland, Oregon using two well performing models, with and without same-scale Gaussian error included.

also shown that more complex models that consider a larger number of variables (i.e., terrain models) do not necessarily make better predictions.

In the end, we advocate renewed rigor and transparency via cross-validated models that use publicly available data to make their conclusions. In terms of future directions, path loss models that make use of active, directed measurements (e.g., [3]) and appropriate statistical methods (e.g., [59]) are promising. We see that picking any single model is precarious. Even with tuning, it is unrealistic to assume any better performance than a straight line fit through 20 measurements. This level of error could have substantial consequences in terms of interference for DSA systems that base their transmission boundaries on *a priori* model predictions.

In our own ongoing and future work, we expect to make measurements at other frequencies of interest (i.e., 700 MHz and 2.5 GHz) in order to begin the development of path loss models that are appropriate for use at widely varying frequencies. We theorize that bounding the error associated with path loss models at different frequencies, and producing appropriate measurement techniques to supplement these models, will be absolutely crucial to future networks that must dynamically make inferences about the channel in an attempt to avoid interference.

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